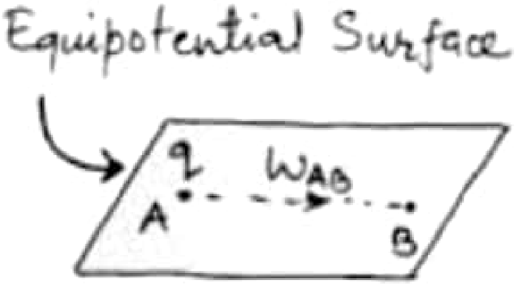
Equipotential Surfaces

That surface at every point of which electric potential is the same is called an equipotential surface.

(i) No work is done on moving the test charge from one point to the other on equipotential surface.



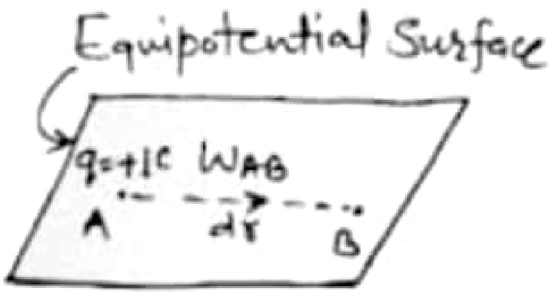
$V_A = V_B$

by $\frac{W_{AB}}{q} = V_B - V_A$

$\frac{W_{AB}}{q} = 0$

$\boxed{W_{AB} = 0}$

(ii) The equipotential surface through a point is normal to the electric field at that point.



$V_A = V_B$

$W_{AB} = 0$ — ①

Also $W_{AB} = \vec{F} \cdot d\vec{r}$

$W_{AB} = F\,dr\cos\theta$

Here $q = +1C$ so that

by $\vec{F} = q\vec{E}$ $\quad \vec{F} = \vec{E}$

Hence $W_{AB} = E\,dr\cos\theta$ — ②

by eq. ① and ②

$E\,dr\cos\theta = 0$

$\theta = 90°$

* The equipotential surface of a single point charge are concentric spherical surfaces centred at the charge.

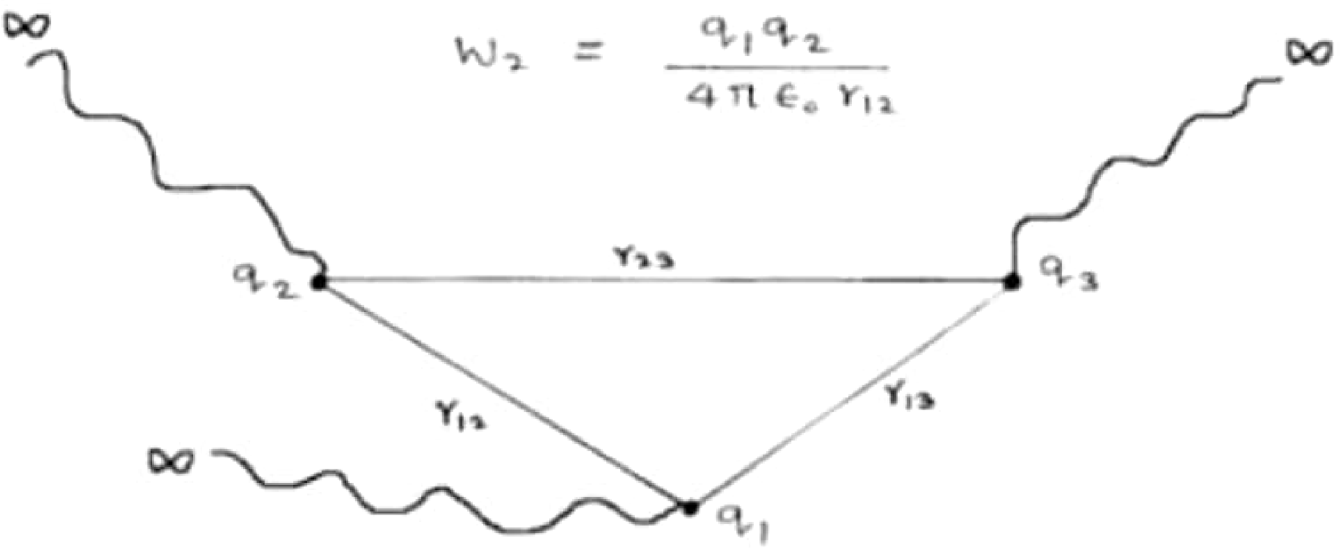
In bringing the first charge q_1 to position $P_1(\vec{r}_1)$ no work is done.

i.e. $W_1 = 0$

When we bring charge q_2 from infinity to $P_2(\vec{r}_2)$ at a distance r_{12} from q_1, work done is

$$W_2 = [\text{Potential due to } q_1] \times q_2$$

$$W_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$



In bringing q_3 from ∞ to $P_3(\vec{r}_3)$, work has to be done against electrostatic forces due to both q_1 and q_2

$$\therefore \quad W_3 = [\text{Potential due to } q_1 \text{ and } q_2] \times q_3$$

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \times q_3$$

$\therefore$ Electrostatic potential energy of a system of three charges

$$U = W_1 + W_2 + W_3 = 0 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

Proceding in this way, we can write electrostatic potential energy of a system of N point charges at $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N$ as -